Injecting Combinatorial Optimization into MCTS: Application to the Board Game boop.

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***Abstract*—Games, including abstract board games, constitute a convenient ground to create, design, and improve new AI methods. In this field, Monte Carlo Tree Search is a popular algorithm family, aiming to build game trees and explore them efficiently. Combinatorial Optimization, on the other hand, aims to model and solve problems with an objective to optimize and constraints to satisfy, and is less common in Game AI. We believe however that both methods can be combined efficiently, by injecting Combinatorial Optimization into Monte Carlo Tree Search to help the tree search, leading to a novel combination of these two techniques. Tested on the board game boop., our method beats 96% of the time the Monte Carlo Tree Search algorithm baseline. We conducted an ablation study to isolate and analyze which injections and combinations of injections lead to such performances. Finally, we opposed our AI method against human players on the Board Game Arena platform, and reached a 373 ELO rating after 51 boop. games, with a 69% win rate and finishing ranked 56th worldwide on the platform over 5,316 boop. players.**

***Index Terms*—Monte Carlo Tree Search, Combinatorial Optimization, Constraint Programming, Board Games**

I. Introduction

During one of the 51 online games opposing our AI agent against a human player, we were asked in the chat "Why researching new AI methods?" It is true that some existing AI methods, like Deep Reinforcement Learning, would certainly defeat any human players at this game, if properly trained.

Although the perspective of making an AI agent with a deep mastery of a game is satisfying, this is not the reason why one does research in Game AI. Research is driven by the quest to push the boundaries of knowledge. This can be done by proposing something new. One way to search for new AI methods is to try combining two existing methods that have never been combined before.

This is what this study aims to do, by combining Monte Carlo Tree Search and Combinatorial Optimization in a way that has been never explored, to the best of our knowledge. This paper actually proposes three possible combinations, or to be more specific, three different injections of Combinatorial Optimization into Monte Carlo Tree Search, to improve performances of the latter. In particular, such combinations can be very profitable on devices with limited computing power, where only a few random playouts can be performed.

The proposed method in this paper is applied on a recent abstract board game called boop. (without capital letters and with a dot.) The main interest of this game is to be simpler than Go or Chess, but deep enough to offer complex strategies. These characteristics motivate its choice to be the testbed of a new method.

II. Background

This section introduces the two combined AI techniques, Monte Carlo Tree Search and Combinatorial Optimization, as well as boop., the board game used as a testbed.

A. Monte Carlo Tree Search

Monte Carlo Tree Search (MCTS) is a family of tree search algorithms relying on the Monte Carlo method, *i.e.,* random samplings.

Originally developed for Go [1], this type of tree search algorithm has been applied successfully to many other board games such as Checkers, Hex and Backgammon, as well as strategy, general and arcade video games [2]–[4]. MCTS has also been combined with Deep Reinforcement Learning to reach state-of-the-art levels at Go, Chess, and Shogi [5], among other games.

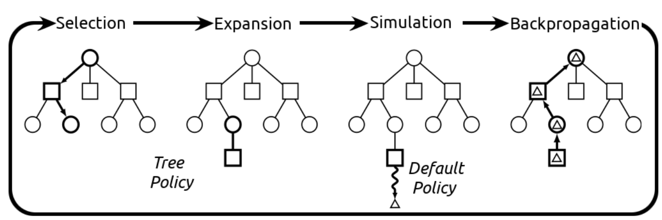


Fig. 1: Steps of the Monte Carlo Tree Search.

MCTS aims to build a game tree of a reasonable width, even with games implying a high branch factor like Go, by focussing on promising branches of the tree. Its principle, depicted in Figure 1, is simple. It consists in the iteration of 4 steps:

1) **Selection**, where a node in the tree is chosen following a given *Tree Policy*.
2) **Expansion**, where a new node is inserted into the tree, by applying a move from the node previously selected.
3) **Simulation**, where moves are successively chosen following a *Default Policy* (usually, random moves) until reaching a stop criterion (usually, the end of the game). Such a series of moves is called a playout or a rollout. In this paper, we call this step the **Playout step**, allowing us to have a simple naming convention for our different agents, as explained in Section V.

4) **Backpropagation**: The state reached after the simulation is evaluated by a function computing a reward, which is backpropagated to its parent node up to the root. This reward will influence the Tree Policy during the Selection step.

After reaching a timeout or a given number of iterations, the MCTS algorithm stops and outputs the move maximizing a given criterion, such as the most visited child of the root node, or the one with the highest reward, etc. The reader can refer to Browne et al's survey [6] for further information on MCTS.

In practice, the Tree Policy for Selection is often determined by computing a Upper Confidence Bound (UCB) function [6]. Applying UCB on MCTS leads to the Upper Confidence bounds applied to Trees (UCT) algorithm [7]. UCT is a special case of MCTS. Although the experimental setup in Section V implements UCT, our method could be applied in principle with any MCTS algorithm. This is why this paper refers to MCTS rather than UCT specifically.

B. Combinatorial Optimization

Combinatorial Optimization is the field aiming to model and solve problems where one must find the optimal combination of discrete variable assignments to maximize or minimize an objective function, while satisfying all given constraints. Several formalisms exist to model such problems: Linear Programming, Answer-Set Programming, etc. In this paper, we model our Combinatorial Optimization problem in a Constraint Programming formalism called Constrained Optimization Problems [8] (COP).

A COP is characterized by the quadruplet (V, D, C, f), where:

- V is the set of decision **variables** of the problem.
- D is the set of **domains**. A domain is the set of values a variable can be assigned to.
- C is the set of **constraints**, forbidding some variable assignment combinations.
- f is the **objective function** to optimize.

There exist two families of algorithms to solve problems modeled in Constraint Programming: Complete and incomplete algorithms. Complete algorithms cover the entire search space by pruning it, and can proof the optimality of a solution. Incomplete algorithms, or meta-heuristics, rely on random moves and heuristics to explore the search space. Although such methods cannot prove the optimality of a solution, they are faster than complete algorithms in practice and can tackle larger problems.

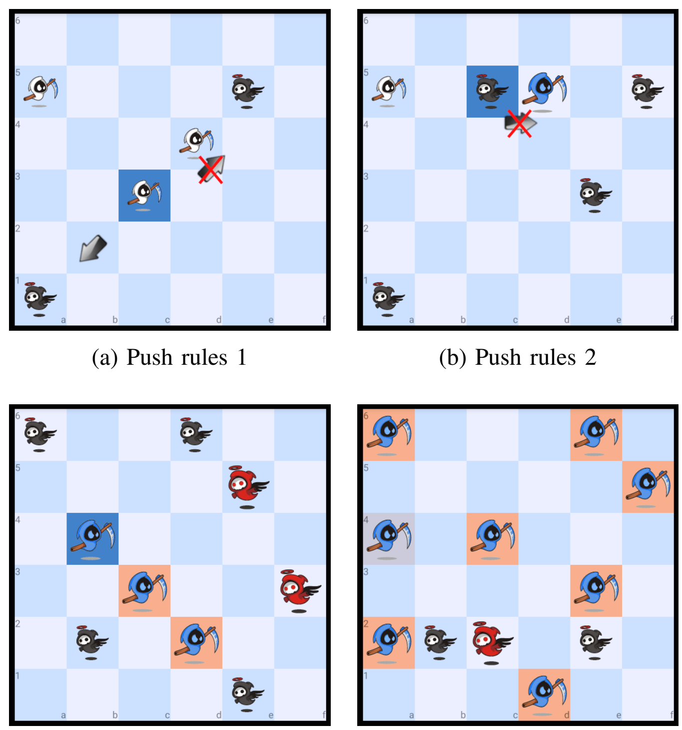
C. boop.

boop. is a board game created by Scott Brady and published in 2022 by Smirk and Dagger Games. It is the commercial version of Gekitai^2 (Gekitai squared), released by Scott Brady for free in 2020 on the website BoardGameGeek. Since both games have exactly the same rules, we will refer to this game only by its commercial name boop.

boop. is a deterministic, fully observable, 2-player game. The rules are simple: Each player has 8 small and 8 large pieces, and starts with a pool of 8 small pieces. Players place alternately one piece from their pool on a free square of the 6×6 board. When a piece is placed, it pushes away all adjacent pieces from one square, except if a piece is blocked by another piece, like depicted in Figure 2a: A white piece has been played in c3 and pushed away a black piece from b2 to a1, but did not push away the white piece in d4 because it is blocked by another piece in e5. Large pieces can push away any other pieces, but small pieces cannot push away large pieces (Figure 2b). When a piece is pushed out of the board, it returns into its player's pool.

When 3 pieces of a player are aligned, they are removed from the board at the end of the player's turn, and return into the player's pool. Small pieces removed that way are promoted to large pieces. If more than 3 pieces are aligned, the player chooses 3 adjacent pieces to remove. If players place their 8 pieces on the board, they can choose one piece to remove from the board. In case this piece is a small one, it is promoted to a large piece.

A player wins the game if he or she has 3 large pieces aligned at the end of his or her turn (Figure 2c), or if 8 large pieces are placed on the board at the end of the player's turn (Figure 2d). There are no tied games in boop.



(a) Push rules 1 (b) Push rules 2

(c) Victory condition 1 (d) Victory condition 2

Fig. 2: Main rules of boop. These images are from a boop. Android app we are developing.

III. Related Work

Many works tried to combine MCTS and Combinatorial Optimization, and more specifically Constraint Programming, but always from a different perspective than ours.

To solve a special case of the Travelling Salesman Problem encountered in the automotive industry, Antuori et al. [11]

combine MCTS and Combinatorial Optimization to improve Combinatorial Optimization solvers by applying MCTS techniques to balance exploration and exploitation of the optimization problem search space. This is a fundamental difference with our work: Where we use Combinatorial Optimization to improve an MCTS method, Antuori et al. use MCTS to improve a Combinatorial Optimization method. A common point is that they replace MCTS playouts with a Deep-First Search method, when we replace them by a series of Combinatorial Optimization problem resolutions, one for each move in the simulation.

In the same manner, the Bandit Search for Constraint Programming (BASCOP) algorithm from Loth et al [12] aims to adapt MCTS to the characteristics of a Constraint Programming search tree. Again, the goal here is to use MCTS to improve the search for a combinatorial problem. Specifically, they designed the MCTS reward function to estimate to each couple (variable, value) a failure score, called *relative failure depth*, then exploited by the solver: Their algorithm guides the Constraint Programming search in the neighborhood of the previous best solution, by exploiting this relative failure depth estimated during the search space exploration.

Goffinet and Ramanujan use MCTS to solve the Maximum Satisfiability Problem (a.k.a. MaxSAT) [13], to balance exploration and exploitation during the search of a SAT solver. They propose the UCTMAXSAT algorithm, where each node in the tree is associated to a variable in the SAT formula, with two possible decisions corresponding to the variable evaluation. Playouts and leaf node evaluations are done by two SAT stochastic local search algorithm runs, one starting by evaluating the expended node by true, the other one by false.

Finally, we can also mention Sabharwa et al.'s work to guide Combinatorial Optimization in Mixed Integer Programming with MCTS [14].

To the best of our knowledge, all works combining Combinatorial Optimization and MCTS methods, like the related works presented above, aim to take advantage of the MCTS capacity to handle the exploration-exploitation dilemma to help Combinatorial Optimization solvers exploring their search space. From that perspective, the work proposed in this paper differs radically from these previous works, combining two methods the other way around, *i.e.,* exploiting Combinatorial Optimization capacities to find optimal solutions under constraints to improve a Monte Carlo Tree Search.

One can also find many Game AI works combining MCTS with heuristics to improve the MCTS method. These works include using heuristics to bias the Tree Policy by replacing or extending the usual UCB function by some heuristics for selection [15], [16], or using heuristics to bias the Default Policy by guiding playouts [17], [18]. Świechowski et al. wrote a good survey about recent MCTS modifications and applications [19]. Our method differs from these works in two aspects:

1) It does not simply use a heuristics, but solve a Combinatorial Optimization problem to bias both the Tree and Default Policies, with the advantages explained in the next paragraph.
2) The Tree Policy is biased without modifying nor replacing the UCB function. Instead, Combinatorial Optimization is used to narrow the number of nodes that can be randomly drawn during the Selection step.

One can notice that the objective function and the constraints of the Combinatorial Optimization model could actually be combined into a unique heuristics by replacing constraints with penalty functions. However, there are three main advantages to bias the Tree and Default Policies by modeling and solving a Combinatorial Optimization problem rather than simply using a heuristics:

1) The heuristics output would not allow to mathematically certify that all constraints are satisfied, unlike solving a COP.
2) Expressing the bias as a COP allows us to take advantage of solvers containing specific mechanisms to exploit the problem structure induced by the constraint network, both with complete solvers (filtering and constraint propagation) and meta-heuristics (constraint-based local search).
3) While using a heuristics, one needs to call it on every possible move. However, constraint solvers do not explore the entire move space: For instance, complete solvers prune the problem search space to avoid visiting subspaces where they determined that solutions are infeasible or suboptimal. Although this feature does not have a strong impact for boop., since the move space of the game is small, this could be very useful for other games and applications with significantly larger move or action spaces.

IV. Mixing MCTS and Combinatorial Optimization

Before describing the Combinatorial Optimization problem, we explain at the beginning of this section how do we combine Combinatorial Optimization and MCTS methods. Then, we give the intuitive idea of the Combinatorial Optimization model in Subsection IV-A, followed by its formal model and some design choices.

Random playouts are a powerful mechanism within MCTS. However, to have a good estimation of the current game state and the value of its possible moves, one must run a significant number of playouts. This is not always easy to do, depending on the hardware: We implemented a vanilla MCTS method within a boop. Android app but quickly realized that the number of playouts we could run on our Android device within a reasonable time was too small to be reliable. Within one second, the device could only run about 80 playouts in average.

This issue can be tackled by replacing playouts with moves that are selected by solving the Combinatorial Optimization problem described in the next Subsection IV-A. Algorithm 1 illustrates how it works. Each move of the playout is randomly drawn among the moves maximizing the Combinatorial Optimization problem (Line 4). After being drawn, a move is simulated to get a new game state (Lines 5 and 6), and an associated reward is computed regarding if the move leads to a terminal game state (Line 8) or not (Line 10). This is repeated until a terminal state is reached or after k moves (while loop at Line 3). Then, the playout stops and returns the cumulative, normalized reward (Line 12). The value of k we choose in practice is discussed in Subsection IV-A. The playout reward is estimated by computing a discounted sum of the normalized scores of the successive k moves, divided by k. Scores are simply the objective function output of the Combinatorial Optimization model and are normalized within the range $[-1, 1]$, such that -1 is the score of a loss and 1 the score of a victory. The discount factor is a parameter d we discuss in Subsection IV-A. We denote by $a = (p, r, c)$ the move placing a piece of type p on the board at row r and column c. Let $a_1, \ldots, a_k$ be the k moves played during a playout. Its playout reward R is estimated by Equation 1

$$R = \frac{1}{k} \sum_{i=1}^{k} d^i . f(a_i) \quad (1)$$

where f is the objective function of the Combinatorial Optimization model. Notice that in Algorithm 1, the *Reward* function on Line 10 corresponds to computing $d^i . f(a_i)$. Since the image of f is $[-1, 1]$, the discount factor is such that $d \leq 1$ holds, and the sum of the k products is divided by k, we have $R \in [-1, 1]$. The playout reward is thus not necessarily $0/1$ or $-1/1$. This is perfectly acceptable for the UCB function, like described in Kocsis and Szepesvári's paper introducing the UCT algorithm: We are here dealing with a P-game tree, that is, "*a minimax tree that is meant to model games where at the end of the game the winner is decided by a global evaluation of the board position where some counting method is employed*" [7], instead of the classic win/loss evaluation.

We also inject the same Combinatorial Optimization problem into the MCTS process to bias the Selection and Expansion steps, as illustrated by Algorithm 2. For the Selection step, the solver is called to pre-select the m best moves regarding the current game state (Line 1). In other words, it selects m nodes among the root's children. Unselected children are masked (Line 2), to prevent the UCB function considering them, forcing to select one of the preselected children (Line 4). This is analogical to invalid action masking in Reinforcement Learning [9], where invalid or poor actions/decisions are masked mostly at the beginning of the learning process, to avoid confused and chaotic situations that are usual during the first iterations, thus shortening the learning. For the Expansion step, the Combinatorial Optimization solver is simply called to find what are the best moves to play, regarding the current game state and excluding the moves that have been already explored (Line 11).

Finally, we set a timeout of 1 second to let our method build and explore the game tree before outputting a move to play. The set of moves with the highest score/visits ratio is computed (Line 19) and the algorithm returns one move randomly drawn from this set, following a uniform distribution (Line 20).

In summary, our method injects Combinatorial Optimization into 3 steps of MCTS: Just before the Selection step (Algorithm 2, Line 1) and during the Expansion step (Algorithm 2, Line 11), to bias to Tree Policy, and during the Playout step,

Algorithm 1: PLAYOUT

```
   Input: A node, an integer k and a game state gs
   Output: The normalized playout score
1  iterations ← 0
2  score ← 0
3  while node is not terminal and iterations < k do
4      best_move ← Random(Solver(gs))
5      node ← Simulate_move(best_move)
6      gs ← Update(gs, node)
7      if node is terminal then
8          score ← score + Terminal_score(node)
9      else
10         score ← score + Reward(node, iterations)
11     iterations ← iterations + 1
12 return score / iterations
```

Algorithm 2: ENHANCED MCTS

```
   Input: A game state gs and a root node
   Output: One of the best estimated moves
1  preselected_moves ← Solver(gs)
2  unselected_mask ← Childs(root) \ preselected_moves
3  while timeout unreached do
       // Select a node in the tree
4      selected ← UCT(unselected_mask)
5      gs ← Update(gs, selected)
6      if selected is terminal then
7          selected.visits ← selected.visits + 1
8          Backprop(selected.parent, selected.score)
9          continue
10     masked_childs ← Childs(selected)
       // Expand the tree with a new node
11     expanded ← Random(Solver(gs, masked_childs))
12     gs ← Update(gs, expanded)
13     expanded.parent ← selected
14     if expanded is not terminal then
15         expanded.score ← Playout(expanded, 20, gs)
16     else
17         expanded.score ← Terminal_score(expanded)
18     Backprop(selected, expanded.score)
19 best_moves ← Best_ratio(preselected_moves)
20 return Random(best_moves)
```

replacing playouts by successive Combinatorial Optimization problem resolutions (Algorithm 1, Line 4), redefining a Default Policy. Despite these modifications, the resulting tree search algorithm is still an MCTS algorithm because all 4 steps are applied, and there are still some randomness in the Playout step: If the Combinatorial Optimization solver finds several optimal solutions, *i.e.,* different moves of the same quality according to the objective function, then one of these moves is randomly selected, following a uniform distribution (Algorithm 1, Line 4, and Algorithm 2, Line 11). Such a situation occurs often in a game: We ran 10 games specifically to evaluate this, and measured it occurs in average 21,352 times per agent and per game.

It is worth noticing that the method presented in this paper focuses on the "move decision-making" in boop., *i.e.,* placing a piece on the board. There is actually a second type of decision players must take in a boop. game: In some occasions, a player has the choice of which pieces to promote. In this work, we handle this decision via a simple heuristics favoring taking pieces on the border of the board. All agents presented in Section V share this heuristics about "promotion decision-making".

The next subsection introduces the tackled Combinatorial Optimization problem.

A. The Combinatorial Optimization model

Before proposing a formal model of the tackled Combinatorial Optimization problem, we first give the intuition behind it. For node pre-selections, expansions, and playouts, the same Combinatorial Optimization problem is solved: Finding a move maximizing the game state score, determined by a given heuristics computed by the objective function, such that the following constraints are satisfied: 1. The piece we play belongs to our pool, 2. Its position is a free square on the board, and 3. The combination (piece type, position) is not a masked move. The two first constraints certify that the move is valid, the last one forces finding a move that does not belong to the set of masked ones. This is necessary for the node pre-selection, where all nodes but m are masked, but also for the expansion, to assure we won't regenerate an existing node.

The following model formally describes the problem presented above:

Variables: $V = \{v_p, v_r, v_c\}$, with v_p the variable deciding the type of piece to play for the move, and v_r, v_c the variables about the row and column number of the move position.
Domains: $D = \{D_{piece}, D_{position}\}$, where $D_{piece} = \{small, large\}$ is the domain of v_p and $D_{position} = \{1, \dots, 6\}$ the domain of v_r and v_c.
Constraints: $C = \{HasPiece(v_p), FreePosition(v_r, v_c), Unmasked(v_p, v_r, v_c)\}$. We formally describe these constraints latter in the section.
Objective function: $f(v_p, v_r, v_c)$ is a heuristics assigning a score to the game state after simulating the move (v_p, v_r, v_c). The exact heuristics function formula is rather long and not easy to display clearly in a paper, but basically, it attributes a score based on the difference between the two players of the number of pieces on the board, on the center and on the border, the difference of large pieces possessed, and if two or three pieces are aligned. This last part of the score differs regarding the type of pieces composing the alignment. The sum of all these is a number in the range [-MAX, MAX]. We divide it by MAX to normalize the outputted score in [-1,1]. The exact heuristics function can be found in the source code[1].

The three constraints of the model can be formally described as follows:

$$HasPiece(v_p) = \begin{cases} true & \text{if } v_p \in \text{player\_pool} \\ false & \text{otherwise} \end{cases}$$

$$FreePosition(v_r, v_c) = \begin{cases} true & \text{if } (v_r, v_c) \in \text{free\_squares} \\ false & \text{otherwise} \end{cases}$$

$$Unmasked(v_p, v_r, v_c) = \begin{cases} true & \text{if } (v_p, v_r, v_c) \notin \text{mask} \\ false & \text{otherwise} \end{cases}$$

To be valid, a variable assignment must be such that all constraints output *true*.

The model contains three parameters, already introduced at the beginning of this section: The number k of moves computed during playouts, the number m of pre-selected nodes and the discount factor d. We did not make an extensive parameter tuning for this study and set their value after some very brief trials. An extensive parameter tuning could probably improve the global performance of our method. This is let as future work. We set $k = 20$, $m = 5$ and $d = 0.9$.

The C++ framework GHOST [10] has been used to model and solve the Combinatorial Optimization problem. It contains a constraint-based local search solver, as well as a backtrack-less, complete solver since its version 3, designed to find all solutions of the tackled problem. We use this complete solver to find and evaluate all possible moves in a given game state.

We can now introduce our experimental setup and results. Two types of experiments have been performed: 1. Section V compares our method with two baselines and with variations of our method, running AI versus AI games. 2. Even if our goal is not to make the best AI agent playing boop., we wanted to evaluate its level against human players. To do so, we played the AI agent against 28 human players in 51 games on the platform Board Game Arena[2]. This is detailed in Section VI.

V. AI versus AI experiments

The goal of this work is to improve MCTS methods by injecting Combinatorial Optimization techniques. A plain, vanilla MCTS method is therefore a natural baseline. Comparing the Combinatorial Optimization-enhanced MCTS with a vanilla MCTS is easy: One just has to disable all Combinatorial Optimization solver calls in the enhanced MCTS to get a vanilla MCTS implementation. Thus, Selection is done considering all children of the root node, and Expansion and Playout are done randomly. We did not give our method a specific name, so we refer to it by MCTS-CO in this section.

This section also compares MCTS-CO with an agent choosing its next move by only calling the heuristics function used in our objective function. This constitutes the second baseline, to test if all improvements reached by MCTS-CO come from the heuristics only, or if it should be attributed to the combination of MCTS and Combinatorial Optimization.

Finally, an ablation study is performed by comparing MCTS-CO with itself when Combinatorial Optimization is enabled

[1] heuristics.cpp

[2] https://boardgamearena.com

or disabled for the Selection, the Expansion and the Playout steps. We denote agents implementing these modifications by MCTS + the first letter of the concerned steps. For instance, MCTS+SP is the MCTS agent injecting Combinatorial Optimization in the Selection and Playout steps. The reader can observe that the agent MCTS-CO corresponds thus to the agent MCTS+SEP.

A. Experimental setup and results

We set a timeout of 1 second for all agents to choose its next move, except for the heuristics agent who does not need any timeout because it does not apply an iterative process: It calls its heuristics function once on each possible move and keeps the move with the highest score, or randomly draws one move among the ones with the highest score.

All experiments have been done through a boop. Android app we are developing, running an Android virtual device on Linux, thus simulating the limited resources of an Android phone compared to a computer. The source code of the Android app, the experimental setup and the results can be found at github.com/richoux/Pobo/releases/tag/0.6.2.

Table I compiles results of 100 games of our MCTS-CO agent against the vanilla MCTS agent, all combinations of Combinatorial Optimization-enhanced MCTS agents, and the heuristics agent. The MCTS-CO agent played half of these games as the first player P1, and the other half as the second player P2.

TABLE I: Number of victories of our MCTS-CO agent versus other agents, being the 1st player P1 or the 2nd player P2. MCTS-CO and its opponent start 50 games each.

MCTS-CO's opponent	MCTS-CO P1	MCTS-CO P2	win rate
Vanilla MCTS	47	49	96%
Heuristics	30	50	80%
MCTS+S	31	46	77%
MCTS+E	40	49	89%
MCTS+P	49	50	99%
MCTS+SE	24	42	66%
MCTS+SP	35	50	85%
MCTS+EP	14	50	64%

We see that MCTS-CO wins 96 games against the vanilla MCTS, over 100 games, showing that injecting Combinatorial Optimization into MCTS leads to very significant improvements. One could argue that these improvements could be obtained with the crafted heuristics function alone, used in the objective function of our Combinatorial Optimization model. This is not the case however, since MCTS-CO also beats 80 times over 100 games the heuristics agent, our second baseline. This shows that the gain of performances comes from the combination of MCTS and Combinatorial Optimization, rather than just the heuristics function alone.

Games against different combinations of Combinatorial Optimization-enhanced MCTS agents allow us to estimate which parts of our methods contribute the most to its improvements. First, one can observe that MCTS-CO significantly outperforms all other Combinatorial Optimization-enhanced MCTS agents. Taken separately, each Combinatorial Optimization injection in the Selection, the Expansion and the Playout step does not bring much compared to the vanilla MCTS agent, with eventually the exception of the MCTS+S agent. It is interesting to observe that, despite being not efficient alone, the Combinatorial Optimization-enhanced Expansion step is a key element while combined with either a Combinatorial Optimization injection in the Selection or the Playout step, as illustrated by the win rate difference of MCTS-CO versus MCTS+SE/EP, and versus MCTS+SP. We can see that injecting Combinatorial Optimization in the Expansion step both greatly keeps up the improvements initiated by MCTS+S, but is also crucial for the Combinatorial Optimization-enhanced Playout step in MCTS+P: Although MCTS+P shows the poorest results among all Combinatorial Optimization-enhanced MCTS agents, MCTS+EP reveals itself to be the best one. We argue that the good synergy between the enhanced Expansion and the enhanced Selection, and in particular between the enhanced Expansion and the enhanced Playout, explains the excellent performance of MCTS-CO against our two baseline agents.

B. Investigating unbalanced results between Player 1 and 2

One can observe from Table I that MCTS-CO's win rate is significantly higher when the agent is playing second, loosing only very few games in that position. There are two possibilities to explain this: 1. The agent is better when playing second for some reasons, 2. The game itself is unbalanced and favors the second player. This would be unusual, since most of the time, the second player is on the contrary disadvantaged in abstract games, but we know that at high level, some boop. players tend to think that starting second is actually a favorable position[3].

Our hypothesis is that the MCTS-CO agent is indeed better when playing as the second player, and that boop. is a correctly balanced game. To check our hypothesis, we run mirror games, *i.e.,* games where both players are the same agent. We made 100 mirror games with the vanilla MCTS agent as a control group, to test if boop. is a well-balanced game and 100 mirror games with MCTS-CO, the heuristics agent, MCTS+S, MCTS+E, and MCTS+P. Results are compiled in Table II.

TABLE II: Results of 100 mirror games.

Agents	P1 wins	P2 wins
Vanilla MCTS	47	53
MCTS-CO	7	93
Heuristics	60	40
MCTS+S	46	54
MCTS+E	46	54
MCTS+P	14	86

Games of the vanilla MCTS agents indicate that there might have some slight advantage for the second player, although statistics over 100 games are not enough to draw solid conclusions. The balance question would deserve a deeper investigation. Moreover, balancing abstract games is notoriously difficult: Chess is considered to be a correctly balanced game, however White playing first has greater chance

[3]From personal communications with highly ranked boop. players on the Board Game Arena platform.

to win: Chessgames.com 2023 statistics indicates that White wins 57.13% of games not finishing with a draw[4]. With 53% of win rates for the vanilla MCTS agent player starting second, it is fair to consider that boop. is a correctly balanced game. This is also confirmed by mirror games with the MCTS+S agent and the MCTS+E agent.

Mirror games of MCTS-CO indisputably shows that the agent is better when playing second. We first thought this was only due to the heuristics function used by the objective function in our model, but this is in contradiction with the results of the heuristics agent's mirror games. Nevertheless, our heuristics function still seems to be the culprit, but in a more complex way, when it is used repeatedly to anticipate the next moves, like in the MCTS+P agent, and of course like in MCTS-CO.

Indeed, playing first at boop. requires a particular attention on the positioning of our own pieces and on decisions to make: The first player needs to "attack" at the right moment, *i.e.,* trying to build alignments of its pieces, neither too early nor too late in the game, whereas the role of the second player consists more to "defend" in early game, trying to break down the first player's formations. This may be an easier role to manage, and that is currently better taken into account by our heuristics function. This bias is amplified by the successive calls of the objective function (and then the heuristics function) in Combinatorial Optimization-enhanced Playout steps, leading to a stronger defense and a weaker attack as well. Taking better early game decisions for the first player would ask an extensive specialization of the heuristics function, which is not the goal of our work in this paper.

VI. AI versus Human experiment

To have a first estimation of the MCTS-CO agent's level against human players, we ask the permission to the Board Game Arena platform for creating an account specifically for this agent[5].

We deployed the following process: After making an announcement on the Board Game Arena forum about our AI agent account, as the Board Game Arena platform recommended us to do, we created boop. games from this account and waited for someone to join. We never joined games created by other players. At the beginning of the game, we used the chat to warn the opponent that he or she is playing against an AI, telling that it is possible to cancel the game without any penalties if he or she is not comfortable with that. Therefore, all opponents were warned they were playing against an AI agent, and all games taken into account for the experiment are games where the opponent agrees to play against the AI. To ensure this, games have been done "manually": We played from the AI agent account on a computer next to an Android tablet running the boop. app implementing MCTS-CO. Then, we reproduced each move from the Board Game Arena opponent on the Android tablet as a human player, and played on Board Game Arena each move decided by the MCTS-CO agent in the app. This way, we acted as a human operator reproducing moves from Board Game Arena to the Android app and from the Android app to Board Game Arena, responding also to eventual questions from opponents on Board Game Arena.

[4]https://www.chessgames.com/chessstats.html

[5]https://boardgamearena.com/player?id=95213950

Board Game Arena is implementing its own ELO rating, which should not be directly compared to Chess ELO rating for instance. Board Game Arena attributes a rank to players according to their ELO rating: Beginner (0 ELO points), Apprentice (1-99), Average (100-199), Good (200-299), Strong (300-499), Expert (500-699), and Master (700+). At the time this experiment was conducted, there were 6 boop. Master players only on the Board Game Arena platform, over 5,316 boop. players.

Our agent played 51 games against 28 players with ELO points from 0 to 865, between the 13th of December, 2023 and the 10th of January, 2024. It won 35 games (69% of win rate), and finished with 373 ELO points (Strong rank), ranked 56th worldwide on the platform. It reached the Strong rank after its 28th game. Figure 3 illustrates the progression of our agent's ELO points. Although its ELO points evolution looks rapid at first glance, it should not be directly compared with the evolutions of human players on Board Game Arena, since many players are likely to discover the game on this platform and then start from a completely beginner level, when our agent played its first games at full strength.

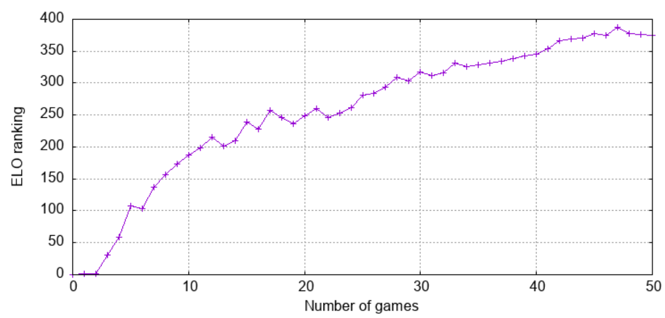


Fig. 3: ELO rating of MCTS-CO on Board Game Arena against human players.

Considering Figure 3, one could think that the agent reached its top performances against humans players, since the ELO points curve seems to converge just below 400 points. We do not think it is the case, though, and believe that it could go further, even maybe reaching the bar of 500 ELO points. The three last games were played against its strongest opponent, a Master player with a 865 ELO rating, ranked 3rd worldwide at that time. Our agent lost these three games and that is what makes the curve flat at the end.

VII. Conclusion and perspectives

We presented in this paper three different injections of Combinatorial Optimization into Monte Carlo Tree Search (MCTS): Just before the Selection step, during the Expansion step, and during the Playout step. While previous works combine MCTS with Combinatorial Optimization solvers to improve them, this is the first time Combinatorial Optimization has been combined with MCTS to improve the latter, to the best of our knowledge. Experimental results show that a Combinatorial

Optimization-enhanced MCTS algorithm greatly outperforms the vanilla MCTS algorithm: In the board game boop., our methods wins 96% of its games against vanilla MCTS, and 80% of its games against an heuristics-based agent, the second baseline, on a virtual device simulating the limited computing resources an Android device may offer, compared to a personal computer. We also did an ablation study allowing us to analyze which Combinatorial Optimization injections are essentials for reaching these performances. From this ablation study, we conclude that injecting Combinatorial Optimization into the Expansion Step is the key stone of our method, performing poorly alone but extremely well while combined with both a Combinatorial Optimization injection into the Selection and the Playout steps.

In parallel of AI versus AI experiments, we also ask the opportunity to the Board Game Arena platform to let our AI agent plays boop. against human players, allowing us to have a rough estimation of its ELO rating. Our agent plays 51 games against 28 opponents of very different skills, winning 69% of its games (35 wins, 16 losses), finishing with a 373 ELO rating (in the "Strong players" class on Board Game Arena) and ranked 56th worldwide on the platform over 5,316 boop. players.

Apart from trivial improvements we could bring to our method and its implementation, such as the tuning of its 3 parameters, an interesting perspective could be modeling and solving a Combinatorial Optimization problem going beyond one-stage decision-making: So far, the Combinatorial Optimization problem we solve aims to find the best move in the current situation, and the combinatorial part of this problem is certainly under-exploited for the solver we use. Tackling k-stage decision-makings, *i.e.,* deciding the move after considering k-1 successive moves, would constitute a great challenge from a combinatorial point of view, for instance by certifying that the opponent does not have any direct winning moves next after our move (unless all our moves are losing moves).

Acknowledgment

We would like to thank the Board Game Arena platform for accepting to let us creating an account and playing games with our MCTS-CO agent against human players, giving us the opportunity to estimate its Board Game Arena ELO rating. We also thank all players from Board Game Arena who accepted to play one or several games against our AI agent.

Figure 1 is a modification of a figure from Wikimedia Commons under CC-BY-SA license. The source can be found at en.wikipedia.org/wiki/File:MCTS-diagram.svg.

[6] https://creativecommons.org/licenses/by/4.0/

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